PHYSICAL REVIEW E, VOLUME 64, 031904

Fluctuation-induced memory retrieval in a pulsed neural network storing sparse patterns with hierarchical correlations

Takashi Kanamaru
Department of Electrical and Electronic Engineering, Tokyo University of Agriculture and Technology, Tokyo 184-8588, Japan

Yoichi Okabe
RCAST, The University of Tokyo, Tokyo 153-8904, Japan

(Received 16 February 2001; published 28 August 2001)

An associative memory in a pulsed neural network composed of the FitzHugh-Nagumo models storing sparse patterns with hierarchical correlations is investigated. The memory patterns composed of 0/1 digits are represented by the synchronous periodic firings of the neurons. It is found that the target pattern and the OR pattern are retrieved individually by controlling the intensity of fluctuations in the system.

DOI: 10.1103/PhysRevE.64.031904 PACS number(s): 87.10.+e, 05.45.−a, 84.35.+i, 07.05.Mh

I. INTRODUCTION

The associative memory in neural networks has been investigated for more than 20 years. In the autocorrelative associative memory model, the bit patterns are stored in the connection coefficients of the network and the stored patterns are retrieved using the neural dynamics. The storage capacity of the network is analyzed extensively by numerous researchers and it is known that the storage capacity for sparse patterns diverges as the firing rate of the pattern approaches zero. The coding of the memory is called ‘‘sparse’’ when the number of excited neurons is much smaller than that of quiescent ones, in other words, the firing rate of the network is small. The existence of sparsely encoded associative memory in the brain is discussed in physiological experiments [1].

On the other hand, it is known that the mixed states of the stored patterns, which are nonlinear superpositions of stored patterns, also become equilibria of the network [2,3]. The typical mixed states are the OR patterns, the AND patterns, and the majority decision mixed states [2]. In a broad sense, such mixed states are the models of the mutually correlated memories that are experimentally observed [4]. In Ref. [3], the dynamics of the network storing memory patterns with hierarchical correlations is analyzed and the mixed states of stored patterns are considered. Such mixed states may be interpreted as unnecessary patterns that accompany with stored patterns, but some researchers relate the stabilization of mixed states of stored patterns with a ‘‘concept formation’’ [5] and discuss the validity of this relation in the physiological experiments [2,6].

Conventionally, the carrier of the information in associative memory models is thought to be the firing rate of a single neuron or an ensemble of neurons. However, neural networks composed of spiking neurons also show the properties of associative memory [7–11], and have attracted considerable attentions in recent years. In those systems, the following models are often used as spiking neurons: the Hodgkin-Huxley model, which describes the dynamics of squid giant axons; the FitzHugh-Nagumo model, which is the reduced model of the Hodgkin-Huxley model; and the leaky integrate-and-fire model, which has an internal state governed by a linear differential equation and a spiking mechanism with a threshold. The couplings among those neurons are accompanied with time delays that model the time lag from the presynaptic neuron to the postsynaptic neuron, and the memory is represented in the spatiotemporal firing pattern of the neurons.

Meanwhile, the physiological environment where neurons operate is thought to have several sources of randomness, such as, thermal noise, stochastic properties of synapses [12,13], and the sum of enormous presynaptic inputs [14], thus the effect of the fluctuation may not be neglected. Generally, stochastic resonance (SR) is a well-known phenomenon where a weak input signal is enhanced by its background fluctuation and observed in many nonlinear systems [15–18]. The theoretical works on SR in a single neuron are performed on the integrate-and-fire model [19], the leaky integrate-and-fire model [20,21], the FitzHugh-Nagumo model [22–24], and the Hodgkin-Huxley model [25]. In those works, it is observed that the output signal-to-noise ratio (SNR) [23,25] or the peak height of the interspike interval distribution [19–22] takes a maximum as a function of the fluctuation intensity. Some physiological experiments reinforce the hypothesis that the neural system utilizes SR to detect weak signals [26–29]. In Ref. [26], Douglass *et al.* investigated sinusoidally stimulated mechanoreceptor cells of a crayfish with additive fluctuations and observed the existence of the optimal fluctuation intensity that maximizes the output SNR. In Ref. [27], Pei *et al.* observed SR in caudal photoreceptor interneurons of a crayfish by intrinsic and not external fluctuations. Recently, theoretical works on SR in spatially extended systems are performed and the roles of fluctuations in neural systems are discussed [30,31]. In Ref. [10], associative memory in a pulsed neural network with fluctuations is considered, and the memory retrieval is induced by the suitable amount of fluctuations. This phenomenon might relate to stochastic resonance.

In the present paper, we treat the sparsely encoded associative memory in a network of the FitzHugh-Nagumo models and consider the effect of fluctuations in the system. In Sec. II, a coupled FitzHugh-Nagumo model and some quantities are defined. In Sec. III, the stored six patterns with hierarchical correlations are defined. In Sec. IV, the results of numerical simulations are presented. It is shown that the target pattern and the OR pattern, which is one of the mixed patterns are retrieved individually by controlling the fluctuation intensity. In Sec. V, theoretical analyses are presented. Conclusions and discussions are given in the final section.

II. ASSOCIATIVE MEMORY OF SPIKING NEURONS

In the following, as a model of associative memory, we treat a coupled FitzHugh-Nagumo (FN) model written as

$$\tau\frac{du_i}{dt}=-v_i+u_i-\frac{u_i^3}{3}+S_i(t)+\eta_i(t)+\sum_{j=1}^{N}J_{ij}\sum_{k\in\kappa(i,j)}\alpha(t-t_j^k-d_p), \tag{1}$$

$$\frac{dv_i}{dt}=u_i-\beta v_i+\gamma, \tag{2}$$

$$\alpha(t)=g_{peak}\frac{t}{t_0}\exp\left(1-\frac{t}{t_0}\right), \tag{3}$$

$$\langle\eta_i(t)\eta_j(t')\rangle=D\delta_{ij}\delta(t-t'), \tag{4}$$

$$\kappa(i,j)\equiv\{k|t_i^f(t)-d_p<t_j^k<t-d_p\}, \tag{5}$$

where $\beta=0.8$, $\gamma=0.7$, $\tau=0.1$, u_i is the fast variable that denotes the membrane potential of the neuron, v_i is the slow variable that represents the refractoriness, $S_i(t)$ is the external input, $\eta_i(t)$ is a Gaussian white noise, t_j^k is the kth firing time of the jth neuron, the firing time is defined as the time when $u_i(t)$ exceeds an arbitrary threshold θ, $t_i^f(t)$ is the latest firing time of the ith neuron at time t, d_p is the uniform time delay, and $\alpha(t)$ is the alpha function with the height g_{peak}. The alpha function [32] models the excitatory postsynaptic potential (EPSP) [33] that has a positive influence on the postsynaptic membrane potential caused by the arrival of the presynaptic signal. The FN model is a general reduced model of the Hodgkin-Huxley equation, which is a qualitative model of the squid giant axon and often used to describe the behaviors of a single neuron. Note that our FN model with the above parameters shows a typical characteristic of a neuron, namely, it has a stable rest state, and with an appropriate amount of disturbance it generates a pulse with a characteristic magnitude of height and width. The time series of $u_1(t)$ for $S_i(t)=0$, $J_{ij}=0$, and $D=0.002$ is shown in Fig. 1. It is observed that two spikes are generated with the help of fluctuations. In the following, the parameters are fixed at $d_p=3$, $g_{peak}=0.5$, $t_0=1$, and $\theta=0$. The validity of these synaptic parameters is discussed later.

The memory patterns stored in the network are defined as follows. First, pattern vectors $\xi^\mu=(\xi_1^\mu,\xi_2^\mu,\dots,\xi_N^\mu)$ $(\mu=1,2,\dots,p)$ are randomly generated according to the probability density

$$P(\xi_i^\mu)=(1-a)\delta(\xi_i^\mu)+a\delta(\xi_i^\mu-1), \tag{6}$$

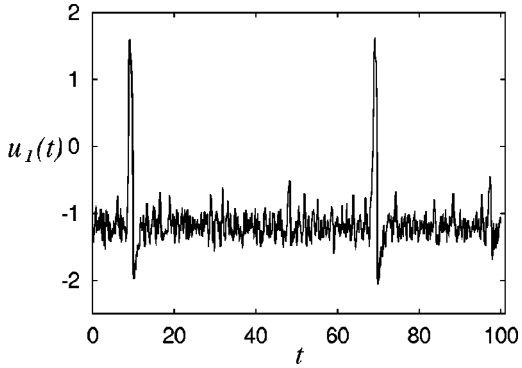

FIG. 1. A typical time series of $u_1(t)$ for $S_i(t)=0$, $J_{ij}=0$, and $D=0.002$. Two spikes are observed.

where $\delta(x)$ denotes the delta function and a $(0\leqslant a\leqslant 1)$ is the average of ξ_i^μ. The dynamics of a network storing the patterns with $a=0.5$ is investigated in Ref. [10]. In the following, sparse patterns with $a=0.1$ are treated.

Next, by applying the bit transformations to ξ_i^μ, p_1 groups of patterns composed of p_2 patterns with overlap b are obtained. Note that the relationship $p=p_1p_2$ holds and the overlap between the patterns ξ and ζ is defined as

$$m(\xi,\zeta)\equiv\frac{1}{Na(1-a)}\sum_{i=1}^{N}(\xi_i-a)(\zeta_i-a). \tag{7}$$

Let us denote the jth pattern in the ith group as $\zeta^{(i,j)}$. The overlap between two patterns that belong to different groups takes zero, namely, the patterns have hierarchical correlations [3] characterized by

$$m(\zeta^{(i,j)},\zeta^{(k,l)})=(b+(1-b)\delta_{jl})\delta_{ik}, \tag{8}$$

$$(1\leqslant i,k\leqslant p_1,\quad 1\leqslant j,l\leqslant p_2).$$

Following Yoshioka and Shiino [9], to make the network store the above p patterns $\zeta^{(k,l)}$ $(1\leqslant k\leqslant p_1, 1\leqslant l\leqslant p_2)$, the connection coefficients J_{ij} are defined as

$$J_{ij}=\frac{1}{Na(1-a)}\sum_{k=1}^{p_1}\sum_{l=1}^{p_2}\zeta_i^{(k,l)}(\zeta_j^{(k,l)}-a). \tag{9}$$

Note that the matrix $J_{ij}\propto\Sigma\zeta_i^{(k,l)}(\zeta_j^{(k,l)}-a)$ is used instead of the usual $J_{ij}\propto\Sigma(\zeta_i^{(k,l)}-a)(\zeta_j^{(k,l)}-a)$ so as not to give negative inputs to the neurons that store 0's, because the FN model can fire even with the negative input due to the rebound effect [34]. It is also noted that Eqs. (1) and (9) indicate that the coupling from the jth neuron to the ith neuron with $\zeta_i^{(k,l)}=\zeta_j^{(k,l)}=1$ has the strength $\sim g_{peak}/Na$. The FN model with our parameters generates a spike when a single EPSP with height ~ 0.1 is injected (data not shown), thus $0.1Na/g_{peak}$ synchronized EPSPs are required to make the postsynaptic neuron to generate a spike. Thus, for large N, it is concluded that the effect of a single presynaptic neuron is weak, which is consistent with the recent physiological observations [14].

The OR pattern vector $\zeta^{OR(k)} = (\zeta_1^{OR(k)}, \zeta_2^{OR(k)}, \ldots, \zeta_N^{OR(k)})$ of the kth group is defined as

$$\zeta_i^{OR(k)} = U\left(\sum_{l=1}^{p_2} \zeta_i^{(k,l)} \right), \tag{10}$$

$$U(x) = \begin{cases} 1 & \text{if } x > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

Generally, the mixed states of stored patterns in the kth group is defined as

$$\zeta_i^{mix(k)} = U\left(\sum_{l=1}^{p_2} \zeta_i^{(k,l)} - \Theta \right), \tag{12}$$

where Θ is an arbitrary threshold [2]. Note that p_2 mixed states are defined by changing Θ, and for $\Theta = 0$ the OR patterns are obtained. It is known that the mixed states of the stored patterns are also memorized in the network [2,3]. The storage capacity of the OR patterns diverges in the sparse limit $a \to 0$, and the storage capacities of other mixed states converge to 0 in the limit $a \to 0$. Thus the OR patterns are "typical" mixed states in the sparse limit. In the following we consider only the OR patterns as the mixes states of stored patterns.

The external input $S_i(t)$ is defined as

$$S_i(t) = x_i U_0 U(t), \quad x_i \in \{0,1\}, \tag{13}$$

$$= \begin{cases} x_i U_0 & \text{if } t \geqslant 0, \\ 0 & \text{otherwise,} \end{cases} \tag{14}$$

where x_i is the binary factor that determines whether the input is injected to the ith neuron or not. In the following, U_0 is fixed at $U_0 = 0.1$, which is so small that each neuron cannot fire without the fluctuation $\eta_i(t)$. Using the binary vector $x = (x_1, x_2, \ldots, x_N)$ of the input, the input overlap $m_{in}^{(k,l)}$, which measures the correlation between the pattern $\zeta^{(k,l)} = (\zeta_1^{(k,l)}, \zeta_2^{(k,l)}, \ldots, \zeta_N^{(k,l)})$ and the external input $S(t) = [S_1(t), S_2(t), \ldots, S_N(t)]$, is defined as

$$m_{in}^{(k,l)} = m(\zeta^{(k,l)}, x). \tag{15}$$

To measure the correlation between the pattern $\zeta^{(k,l)}$ and the time series $u_i(t)$ $(i = 1, 2, \ldots, N)$, $u_i(t)$ is transformed into the binary series $y_i(t) \in \{0,1\}$ written as

$$y_i(t) = \begin{cases} 1 & \text{if } t < t_i^f(t) + \Delta_d, \\ 0 & \text{otherwise,} \end{cases} \tag{16}$$

where the parameter Δ_d is set close to the characteristic width of the output pulse, and $\Delta_d = 4$ is used in the following. Then the overlap between the state of the network and the pattern ζ is defined as

$$m_{out} = \frac{1}{Nf(1-f)} \sum_{i=1}^{N} (y_i - f)(\zeta_i - f), \tag{17}$$

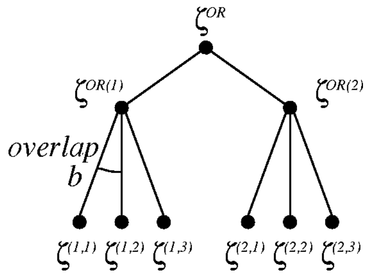

FIG. 2. A schematic diagram of six pattern vectors.

$$f = \frac{1}{N} \sum_{i=1}^{N} \zeta_i . \tag{18}$$

In the following sections, we demonstrate that the target pattern $\zeta^{(1,1)}$ and the OR pattern $\zeta^{OR(1)}$ of the first group can be retrieved individually by controlling the fluctuation intensity D.

III. DEFINITION OF SIX PATTERN VECTORS

In the following, only the case with $a = 0.1$, $p_1 = 2$, and $p_2 = 3$ is considered for simplicity. The larger the number of neurons becomes, the more groups or patterns can be stored, and similar results shall be obtained, but we must perform the numerical experiments with relatively small N, p_1, and p_2 mainly because of the limited computational power.

A schematic diagram of six patterns $\zeta^{(k,l)}$ $(k = 1,2, l = 1,2,3)$ is shown in Fig. 2. In this section, we define these six pattern vectors. First, let us denote the set of indices of neurons that store 1's in the pattern $\zeta^{(k,l)}$ by

$$G(k,l) = \{ i \mid \zeta_i^{(k,l)} = 1, 1 \leqslant i \leqslant N \}. \tag{19}$$

The sets $G(1,l)$ in the space of neuron indices are shown in Fig. 3. Note that the number of elements of the set $G(k,l)$ is

$$|G(k,l)| = \sum_{i=1}^{N} \zeta_i^{(k,l)} = Na. \tag{20}$$

Because the overlap between the patterns $\zeta^{(k,l_1)}$ and $\zeta^{(k,l_2)}$ $(l_1 \neq l_2)$ is b, the number of elements of the intersection of $G(k,l_1)$ and $G(k,l_2)$ is calculated to be

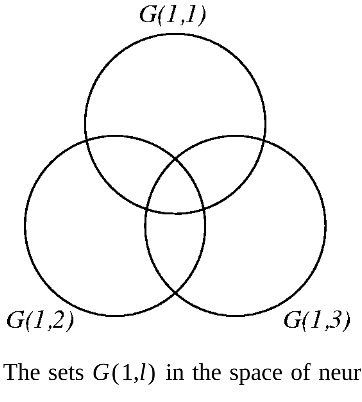

FIG. 3. The sets $G(1,l)$ in the space of neuron indices.

$$|G(k,l_1)\cap G(k,l_2)|=\sum_{i=1}^{N}\zeta_i^{(k,l_1)}\zeta_i^{(k,l_2)}, \tag{21}$$

$$=Na(a+b-ab)\quad (l_1\neq l_2). \tag{22}$$

Though the number of elements of the set $G(k,1)\cap G(k,2)\cap G(k,3)$ is not determined by the parameters a and b, we assume that the probability that the element of $G(k,1)\cap G(k,2)$ belongs to $G(k,1)\cap G(k,2)\cap G(k,3)$ is identical with the probability that the element of $G(k,1)$ belongs to $G(k,3)$. Under such an assumption, the number of elements of the set $G(k,1)\cap G(k,2)\cap G(k,3)$ is calculated to be

$$|G(k,1)\cap G(k,2)\cap G(k,3)|=Na(a+b-ab)^2. \tag{23}$$

Thus the number of elements of the set $G(1,1)\cup G(1,2)\cup G(1,3)$ is $Na[3-3(a+b-ab)+(a+b-ab)^2]$, and we denote it as N_{all} in the following.

Without loss of generality, the pattern $\zeta^{(1,1)}$ can be defined as

$$\zeta_i^{(1,1)}=\begin{cases}1, & 1\leqslant i\leqslant Na=24\\ 0 & \text{otherwise,}\end{cases} \tag{24}$$

and $\zeta^{(1,2)}$ and $\zeta^{(1,3)}$ are defined so that the OR pattern $\zeta^{OR(1)}$ of the first group satisfies

$$\zeta_i^{OR(1)}=\begin{cases}1, & 1\leqslant i\leqslant N_{all}=62\\ 0 & \text{otherwise.}\end{cases} \tag{25}$$

The patterns $\zeta^{(2,l)}$ $(l=1,2,3)$ are determined randomly so that they satisfy Eq. (8).

IV. FLUCTUATION-INDUCED PATTERN SELECTION

Under the above configurations, numerical simulations are performed for $N=240$, $a=0.1$, $b=0.07$, and $m_{in}^{(1,1)}=0.6$. At the time $t=0$, the variables u_i and v_i are set around the equilibrium, namely, $u_i\simeq-1.2$ and $v_i\simeq-0.63$.

The firing times of all the neurons for the fluctuation intensity $D=0.001$ are shown in Fig. 4(a). It is observed that the neurons that store 1's in the pattern $\zeta^{(1,1)}$ start to fire periodically at $t\simeq50$. Let us denote the overlap between the state of the network and the pattern $\zeta^{(1,1)}$ as $m^{(1,1)}$, and the overlap between the state of the network and the OR pattern $\zeta^{OR(1)}$ of the first group as $m^{OR(1)}$. The time series of overlaps $m^{(1,1)}$ and $m^{OR(1)}$ are shown in Fig. 4(b). The overlap $m^{(1,1)}$ almost reaches 1 at $t\simeq50$, thus the retrieval of pattern $\zeta^{(1,1)}$ is successful.

The result of the numerical simulation for $D=0.017$ is shown in Fig. 5. At small t, the pattern $\zeta^{(1,1)}$ is retrieved, but at $t\simeq80$, it is observed that $m^{OR(1)}$ exceeds $m^{(1,1)}$, thus in this case the OR pattern $\zeta^{OR(1)}$ is successfully retrieved.

From the above results, it can be concluded that the target pattern is retrieved for the small fluctuation intensity, and the OR pattern is retrieved for the moderate fluctuation intensity.

In Fig. 6, the asymptotic values of overlaps $m^{(1,1)}$ and $m^{OR(1)}$ are plotted against the fluctuation intensity D. It is observed that the overlap $m^{(1,1)}$ takes a maximum at $D\simeq0.001$. This phenomenon is similar to so-called stochastic resonance, where a weak input signal is enhanced by its background fluctuations. It is also observed that the overlap $m^{OR(1)}$ takes a maximum at $D\simeq0.0017$. Thus it can be concluded that the target pattern and the OR pattern can be retrieved individually by controlling the fluctuation intensity. In other words, a pattern selection is induced by the fluctuations in the system. While the fluctuation-induced pattern retrieval has already been reported in Ref. [10], the present result indicates that the fluctuation can play more functional roles such as pattern selection.

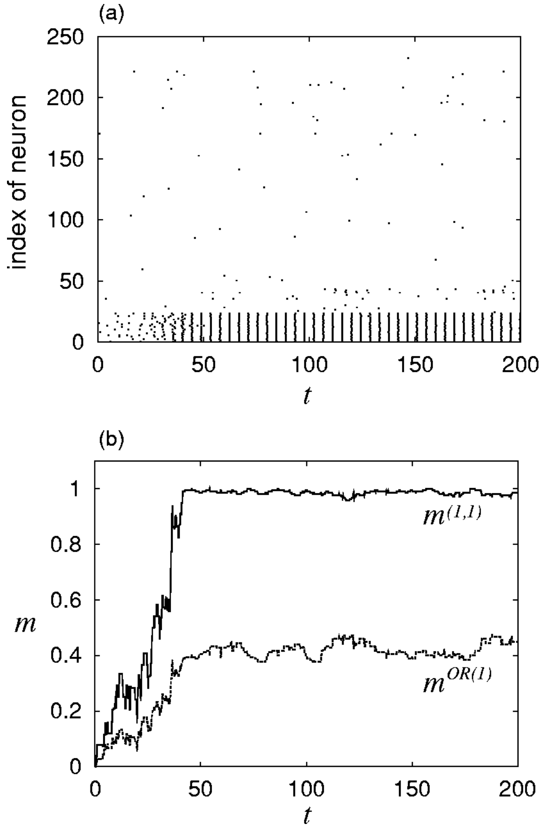

FIG. 4. The result of numerical simulation, (a) the firing times of all the neurons and (b) the overlaps $m^{(1,1)}$ and $m^{OR(1)}$ for $N=240$, $b=0.07$, and $D=0.001$. The retrieval of the pattern $\zeta^{(1,1)}$ is successful.

If the fluctuations are realized by thermal noise, it shall be difficult to control their intensities in the biological environment. On the other hand, it is known that the sum of enormous random EPSPs can behave like fluctuations in the postsynaptic neuron [14,35]. If the sum of EPSPs from the other subnetwork in the brain behaves like fluctuations in the associative network, it might be natural and realizable to control their intensities. It is also known that such a fluctuationlike input can induce SR in a single neuron model [36–41].

To realize the fluctuation-induced pattern selection, the overlap b between the patterns in the identical group is also important. The asymptotic values of the overlaps as a function of the fluctuation intensity D for $b=0$ and 0.1 are shown in Figs. 7 and 8, respectively. For $b=0$, it is observed that

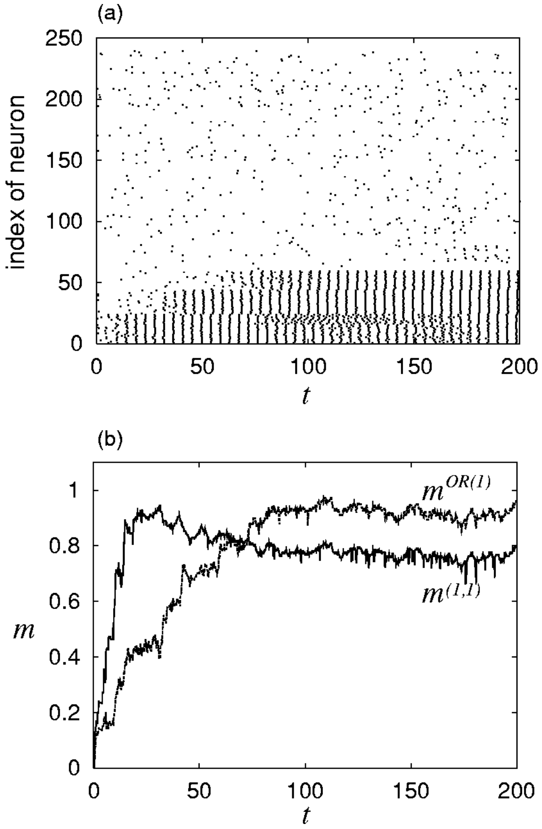

FIG. 5. The result of numerical simulation, (a) the firing times of all the neurons and (b) the overlaps $m^{(1,1)}$ and $m^{OR(1)}$ for $N = 240$, $b = 0.07$, and $D = 0.0017$. The retrieval of the OR pattern $\zeta^{OR(1)}$ is successful.

only the target pattern $\zeta^{(1,1)}$ is successfully retrieved and the retrieval of $\zeta^{OR(1)}$ fails. For $b = 0.1$, it is observed that only the OR pattern $\zeta^{OR(1)}$ is successfully retrieved and the retrieval of $\zeta^{(1,1)}$ fails. Thus it is concluded that the overlap b between the patterns in the identical group is important to retrieve both the target pattern and the OR pattern. In biological systems, such a regulation of b might be realized during the learning process of patterns [4].

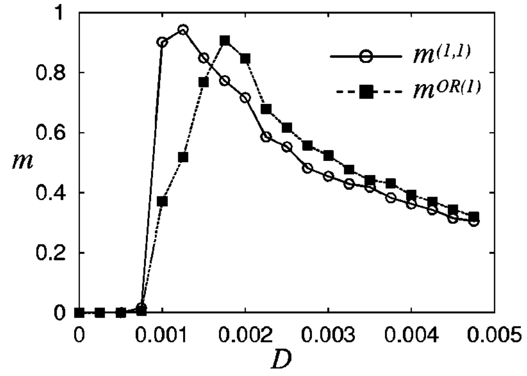

FIG. 6. The asymptotic values of the overlaps $m^{(1,1)}$ and $m^{OR(1)}$ as a function of the fluctuation intensity D for $N = 240$ and $b = 0.07$. Each overlap is numerically obtained by averaging the value over $150 \leqslant t \leqslant 200$.

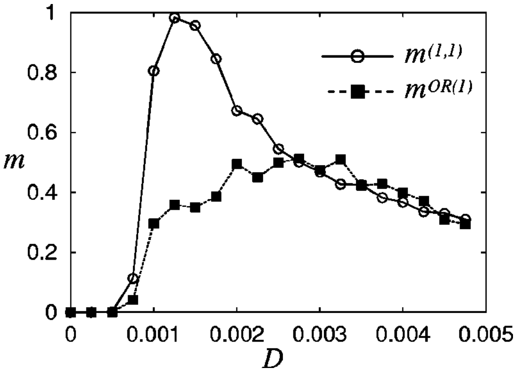

FIG. 7. The asymptotic values of the overlaps $m^{(1,1)}$ and $m^{OR(1)}$ as a function of the fluctuation intensity D for $N = 240$ and $b = 0$. Each overlap is numerically obtained by averaging its value over $150 \leqslant t \leqslant 200$. Only the target pattern is successfully retrieved.

As shown in Figs. 4 and 5, there is a characteristic time required for the retrieval of patterns. Let us define the periods to retrieve the patterns $\zeta^{(1,1)}$ and $\zeta^{OR(1)}$ as $T^{(1,1)}$ and $T^{OR(1)}$, respectively. The dependences of $T^{(1,1)}$ and $T^{OR(1)}$ on the fluctuation intensity D for $b = 0.07$ are shown in Fig. 9. It is observed that $T^{(1,1)}$ and $T^{OR(1)}$ diverge at $D = 0.0009$ and 0.0013, respectively. It is because the retrieval of patterns is realized by the saddle-node bifurcation with the parameter D. This dynamics is treated in the next section.

V. THEORETICAL ANALYSIS OF FLUCTUATION-INDUCED PATTERN SELECTION

In this section, we give a qualitative explanation for the fluctuation-induced pattern selection. In the following, the system with $p_1 = 1$ and $p_2 = 3$, namely, a network that stores three patterns $\zeta^{(1,l)}$ $(l = 1,2,3)$ with overlap b is considered for simplicity. The external input is injected only to the neurons in the set $G(1,1)$, namely, $x = \zeta^{(1,1)}$. In the following, we treat only the dynamics of the neurons in the set $G(1,1) \cup G(1,2) \cup G(1,3)$ for simplicity.

As shown in Fig. 10, two variables z_n and σ_n are defined as the number of the firing neurons and the standard deviation of the firing times in the narrow time interval, respectively. The width of the time interval is set at a value around

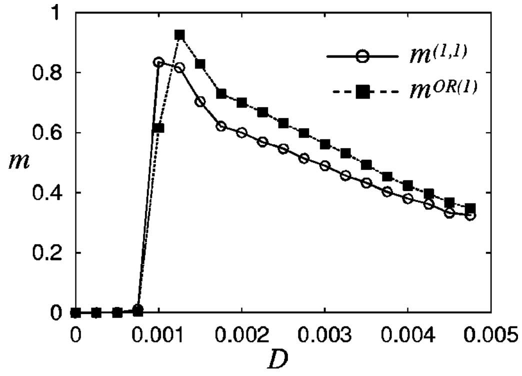

FIG. 8. The asymptotic values of the overlaps $m^{(1,1)}$ and $m^{OR(1)}$ as a function of the fluctuation intensity D for $N = 210$ and $b = 0.1$. Each overlap is numerically obtained by averaging its value over $150 \leqslant t \leqslant 200$. Only the OR pattern is successfully retrieved.

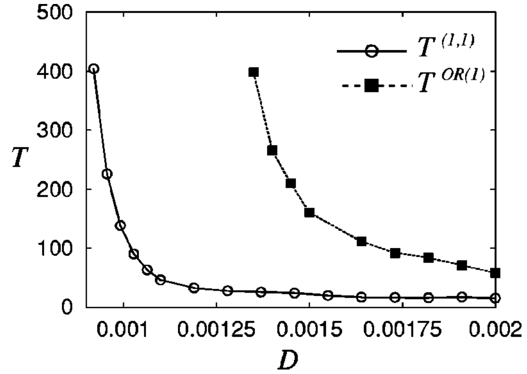

FIG. 9. The dependences of $T^{(1,1)}$ and $T^{OR(1)}$ on the fluctuation intensity D for $b = 0.07$. The data are obtained by taking the mean values of 100–200 samples. The number of samples depends on the fluctuation intensity D.

the time delay of the network, namely, the period of the periodic firing. In the following, the discrete-time dynamics of (σ_n, z_n) $(n = 0,1,2,\ldots)$ is considered. This analysis is similar to the theoretical analysis of the propagation of synchronized spikes in the feedforward synfire chain [42]. The numerical simulations are performed for $Na = 100$, $p_1 = 1$, and $p_2 = 3$.

The numerically obtained flows in the (σ, z) plane for $D = 0.0005$ are shown in Fig. 11(a). Note that the number z_n of the firing neurons is normalized by the number of the neurons that store 1's in the pattern $\zeta^{(1,1)}$, namely, Na. Three attractors in the (σ, z) plane are observed. One is (0,0), and the rest are the attractors corresponding to $\zeta^{(1,1)}$ and $\zeta^{OR(1)}$. The reason why the attractor denoting $\zeta^{OR(1)}$ is not a single node but a line $z \simeq 2.6$ is given later. When the number of the initially firing neurons is sufficiently small, namely, $z_0 \simeq 0$, it is observed that (σ_n, z_n) converges to (0,0). In other words, the memory retrieval fails for $z_0 \simeq 0$ with $D = 0.0005$ because almost all the neurons cannot fire with this fluctuation intensity. Note that the system cannot cross the dotted curve about $z_n \simeq 0.2$ shown in Fig. 11(a), which shows the boundary of the basins for the attractors (0,0) and $\zeta^{(1,1)}$. This boundary seems to be the stable manifold of a saddle at about (σ, z)

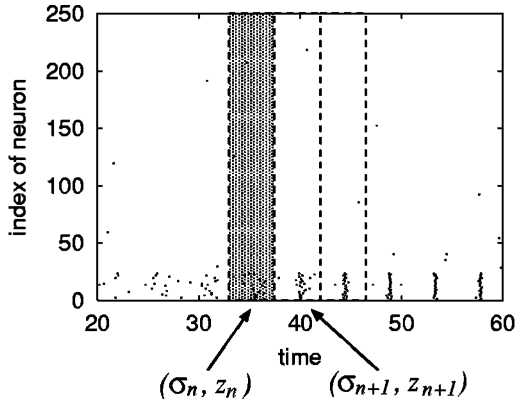

FIG. 10. The definition of the two variables z_n and σ_n. The width of the time interval is set at a value around the time delay of the network, namely, the period of the periodic firing.

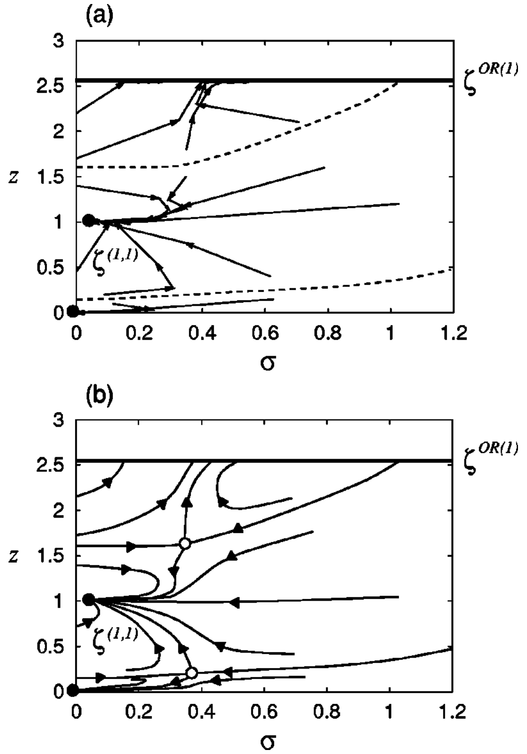

FIG. 11. The flows in the (σ, z) plane for $D = 0.0005$. (a) The numerically obtained flows and (b) the schematic flows deduced from (a).

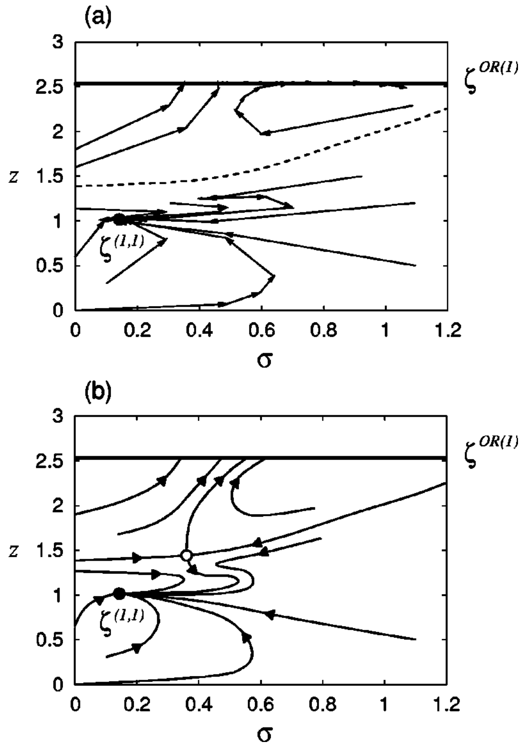

FIG. 12. The flows in the (σ, z) plane for $D = 0.0012$. (a) The numerically obtained flows and (b) the schematic flows deduced from (a).

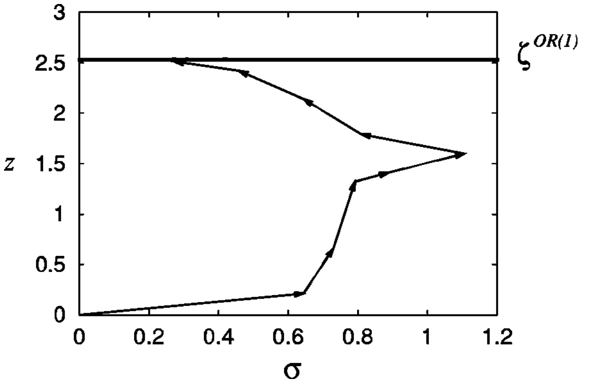

FIG. 13. The numerically obtained flows in the (σ,z) plane for $D=0.002$.

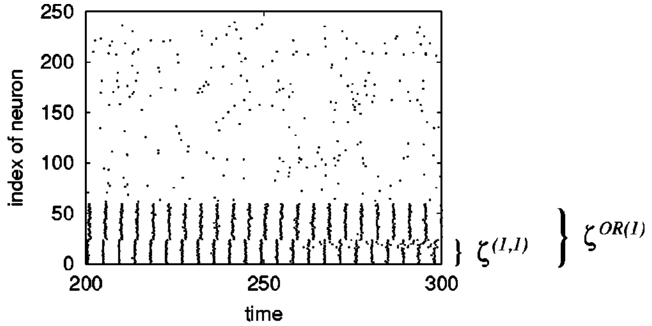

FIG. 14. The firing times of all the neurons in the system for $N=240$, $b=0.07$, and $D=0.0017$. It is observed that the neurons in the range from 1 to 24 fire slightly earlier than the other neurons.

$=(0.4,0.2)$ shown in Fig. 11(b).

The numerically obtained flows in the (σ,z) plane for $D=0.0012$ are shown in Fig. 12(a). The attractor at about (0,0) disappears because of a saddle-node bifurcation, thus the system initially put at (0,0) converges to the attractor at about (0.15,1), which denotes the pattern $\zeta^{(1,1)}$.

The numerically obtained flows in the (σ,z) plane for $D=0.002$ are shown in Fig. 13. The attractor that denotes the pattern $\zeta^{(1,1)}$ disappears because of a saddle-node bifurcation again, thus the system initially put at (0,0) converges to the line at $z\simeq 2.6$ that denotes the pattern $\zeta^{OR(1)}$.

As previously noted, the pattern $\zeta^{OR(1)}$ cannot be denoted by a single node in the (σ,z) plane, because all the neurons that store 1's in $\zeta^{OR(1)}$ cannot synchronize each other as shown in Fig. 14. The step inputs are injected to the neurons that store 1's in the pattern $\zeta^{(1,1)}$, thus they fire slightly earlier than the other neurons.

VI. CONCLUSIONS AND DISCUSSIONS

The associative memory in a pulsed neural network storing sparse patterns with hierarchical correlations is investigated. The stored memory patterns composed of 0/1 digits are represented by the synchronous periodic firings in the network. It is found that the retrieval of the target pattern is achieved by adding fluctuations to the system. This phenomenon is similar to so-called stochastic resonance, where a weak input signal is enhanced by its background fluctuations. Though there is no time-dependent input in our network, the mechanism of associative memory is driven and enhanced by fluctuations. Besides the target pattern, the OR pattern that is the nonlinear superposition of the three patterns that belong to the identical group is also retrieved with the help of fluctuations and its optimal fluctuation intensity is larger than that of the target pattern. Thus the target pattern and the OR pattern are retrieved individually by controlling the fluctuation intensity, in other words, a fluctuation-induced pattern selection takes place. The theoretical analysis of the above results is also presented, and it is found that the fluctuation-induced pattern selection is realized by the successive saddle-node bifurcations parameterized by the fluctuation intensity.

The OR pattern may be interpreted as unnecessary patterns that accompany with stored patterns, but some researchers relate the stabilization of mixed states of stored patterns with a "concept formation" [5], and discuss the validity of this relation in the physiological experiments [2,6]. If the OR pattern is meaningful in the information processing, the above results suggest that the fluctuations in the system might play significant roles in the brain.

There are several sources of randomness in the physiological environment where neurons operate, such as, thermal noise, stochastic properties of synapses [12,13], and the sum of enormous presynaptic inputs [14]. It is difficult to control the intensity of thermal noise, but, if the stochasticity of synapses or the sum of enormous presynaptic inputs act as fluctuations, it might be natural to control their intensities by the synaptic potentiation and depression. It is known that stochastic resonance also takes place by fluctuationlike presynaptic inputs [36–41].

Recently, a psychological experiment revealed that a moderate magnitude of acoustic noise minimizes the response time to retrieve memories [43]. Though our results are based on numerical simulations, this experiment may relate to the fluctuation-induced memory retrieval and suggests that fluctuations may play a significant role in the real brain.

ACKNOWLEDGMENTS

The author (T.K.) is grateful to Professor Takehiko Horita and Professor Kazuyuki Aihara for their stimulating and useful discussions.

---

[1] M. Okada, Neural Networks **9**, 1429 (1996).
[2] T. Kimoto and M. Okada, e-print cond-mat/9911187.
[3] K. Toya, K. Fukushima, Y. Kabashima, and M. Okada, e-print cond-mat/9909276.
[4] Y. Miyashita, Nature (London) **335**, 817 (1988).
[5] S. Amari, Biol. Cybern. **26**, 175 (1977).
[6] Y. Sugase, S. Yamane, S. Ueno, and K. Kawano, Nature (London) **400**, 869 (1999).
[7] W. Gerstner, R. Ritz, and J.L. van Hemmen, Biol. Cybern. **69**, 503 (1993).

[8] W. Maass and T. Natschläger, Network **8**, 355 (1997).
[9] M. Yoshioka and M. Shiino, Phys. Rev. E **58**, 3628 (1998).
[10] T. Kanamaru and Y. Okabe, Phys. Rev. E **62**, 2629 (2000).
[11] H. Hasegawa, e-print cond-mat/0007198.
[12] N.A. Hessler, A.M. Shirke, and R. Malinow, Nature (London) **366**, 569 (1993).
[13] C. Rosenmund, J.D. Clements, and G.L. Westbrook, Science **262**, 754 (1993).
[14] M.N. Shadlen and W.T. Newsome, Curr. Opin. Neurobiol. **4**, 569 (1994).
[15] F. Moss, D. Pierson, and D. O'Gorman, Int. J. Bifurcation Chaos Appl. Sci. Eng. **4**, 1383 (1994).
[16] M.I. Dykman, D.G. Luchinsky, R. Mannella, P.V.E. McClintock, N.D. Stein, and N.G. Stocks, Nuovo Cimento D **17**, 661 (1995).
[17] K. Wiesenfeld and F. Jaramillo, Chaos **8**, 539 (1998).
[18] L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).
[19] A.R. Bulsara, S.B. Lowen, and C.D. Rees, Phys. Rev. E **49**, 4989 (1994).
[20] A.R. Bulsara, T.C. Elston, C.R. Doering, S.B. Lowen, and K. Lindenberg, Phys. Rev. E **53**, 3958 (1996).
[21] T. Shimokawa, K. Pakdaman, and S. Sato, Phys. Rev. E **59**, 3427 (1999).
[22] A. Longtin, J. Stat. Phys. **70**, 309 (1993).
[23] K. Wiesenfeld, D. Pierson, E. Pantazelou, C. Dames, and F. Moss, Phys. Rev. Lett. **72**, 2125 (1994).
[24] A. Longtin and D.R. Chialvo, Phys. Rev. Lett. **81**, 4012 (1998).
[25] S. Lee and S. Kim, Phys. Rev. E **60**, 826 (1999).
[26] J.K. Douglass, L. Wilkens, E. Pantazelou, and F. Moss, Nature (London) **365**, 337 (1993).
[27] X. Pei, L.A. Wilkens, and F. Moss, J. Neurophysiol. **76**, 3002 (1996).
[28] B.J. Gluckman, T.I. Netoff, E.J. Neel, W.L. Ditto, M.L. Spano, and S.J. Schiff, Phys. Rev. Lett. **77**, 4098 (1996).
[29] B.J. Gluckman, P. So, T.I. Netoff, M.L. Spano, and S.J. Schiff, Chaos **8**, 588 (1998).
[30] J.J. Collins, C.C. Chow, and T.T. Imhoff, Nature (London) **376**, 236 (1995).
[31] T. Kanamaru, T. Horita, and Y. Okabe, Phys. Lett. A **255**, 23 (1999).
[32] *Methods in Neuronal Modeling*, edited by C. Koch and I. Segev (The MIT Press, Cambridge, 1989).
[33] J. G. Nicholls, A. R. Martin, B. G. Wallace, and P. A. Fuchs, *From Neuron to Brain*, 4th ed. (Sinauer Associates, Sunderland, MA, 2001).
[34] W.K. Luk and K. Aihara, Biol. Cybern. **82**, 455 (2000).
[35] S. O. Rice, in *Selected Papers on Noise and Stochastic Processes*, edited by N. Wax (Dover, New York, 1954).
[36] X. Godivier and F. Chapeau-Blondeau, Europhys. Lett. **35**, 473 (1996).
[37] F. Chapeau-Blondeau, X. Godivier, and N. Chambet, Phys. Rev. E **53**, 1273 (1996).
[38] G. Mato, Phys. Rev. E **58**, 876 (1998).
[39] G. Mato, Phys. Rev. E **59**, 3339 (1999).
[40] Y. Sakumura and K. Aihara, in *Proceedings of the Fifth International Conference on Neural Information Processing*, edited by S. Usui and T. Omori (Ohmsha, Tokyo, 1998), p. 951.
[41] J. Feng and B. Tirozzi, Phys. Rev. E **61**, 4207 (2000).
[42] M. Diesmann, M. Gewaltig, and A. Aertsen, Nature (London) **402**, 529 (1999).
[43] M. Usher and M. Feingold, Biol. Cybern. **83**, L11 (2000).